What the Casimir-Effect really is telling about Zero-Point Energy

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Casimir predicted an attractive force between metallic surfaces, which according to his model is caused by the zero-point oscillations of the quantized electromagnetic field. Following a suggestion by Casimir, we assume in this article that the reflection spectra of metals are at least approximately identical for the reflection of photons and the reflection of zero-point oscillations. It is shown that this assumption turns Casimir's argument to the exact opposite: The observed Casimir-force positively proves, that the electromagnetic field's zero-point energy does *not* exert forces onto metallic surfaces, if that assumption on reflectivity should be correct. We add reasons for the assertion, that Casimir's assumption on reflectivity is probably wrong, and that an improved assumption is still disproving Casimir's model.

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1. Zero-point energy

When Heisenberg discovered quantum mechanics in 1925 [1], harmonic and anharmonic oscillators were the first systems, to which he applied his novel formalism. His equations led to the quantized energy-spectrum

$$E = \left(n + \frac{1}{2}\right) h\nu \quad \text{with } n = 0, 1, 2, 3, \dots \tag{1}$$

The zero-point energy $h\nu/2$ per degree of freedom had been established experimentally already before due to the analysis of the vibrational spectra of molecules [2]. Further experimental evidence

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for the physical existence of zero-point energy arose in the following years for example from the scattering of X-rays by crystals at low temperature [3] and from the observation that ^{4}He stays liquid at normal pressure even near $T = 0$ [4]. Thus the physical existence of zero-point energy in systems with a finite number of degrees of freedom was well established already in the early years of quantum theory both experimentally and theoretically.

In contrast, there were at that time — and still are by today — severe doubts regarding the physical existence of the infinitely large zero-point energy resulting from the quantization of continuous fields. It was not only the infinitely large value of the energy, which caused concern; that could be reduced to finite values by some appropriate regularization method. But there were simply no positive indications of it's existence known from observations. Quite the contrary: Zero-point energy, like any form of energy, should gravitate and thus, due to it's huge value, result in observable curvature of the intergalactic space, provided that general relativity theory is correct. Pauli made an estimation of the curvature of space, which was to be expected due to the electromagnetic field's zero-point energy. "The result was, that the radius of the universe (if short wavelengths are cut-off at the classical electron radius) 'would not even reach to the moon'." [5, page 842]

In an 1928 article on the quantization of the electromagnetic field, Jordan and Pauli concluded (my translation): "It seems to us, that several considerations are indicating, that — in contrast to the eigen-oscillations in the crystal grid (where both theoretical and empirical reasons are indicating the existence of a zero-point energy) — no reality can be assigned to that 'zero-point energy' $h\nu/2$ per degree of freedom in case of the eigen-oscillations of the radiation. As one is dealing with regard to the latter with strictly harmonic oscillators, and as that 'zero-point radiation' can neither be absorbed nor scattered nor reflected, it seems to elude, including it's energy or mass, any method of detection. Therefore it may be the simplest and most satisfactory conception, that in case of the electromagnetic field that zero-point radiation does not exist at all." [6, page 154]

2. The Casimir-effect

By a publication of Casimir [7] in 1948, the opinion of Jordan and Pauli seemed to be refuted. Casimir's computation (on which we will dwell below) resulted in an attractive force

$$F_{\text{Casimir}} = -\frac{\pi^2 \hbar c}{240} \frac{XY}{D^4} = -1.3 \cdot 10^{-9}\text{N} \cdot \frac{XY/\text{mm}^2}{D^4/\mu\text{m}^4} \tag{2}$$

between two parallel metallic plates with area XY, which are separated by a gap of width D. This force is caused in Casimir's model by the quantized electromagnetic field's zero-point energy, which thus should be observable in the laboratory. In the following decades, the Casimir-force has many times been observed experimentally, and it was confirmed that equation (2) is — at least approximately — correct [8].

As (2) is based on the assumption of perfectly reflecting metals, experimentalists can achieve good fits of their results only if they apply corrections to this formula, considering the finite conductivity of the metals used in their experiments [8]. The reflection coefficients used in these fits are the measured reflection coefficients for photons. Combining this fact with the assumption, that the Casimir-force is allegedly caused by zero-point oscillations, one must assume

$$0 < R_{\text{phot}}(\lambda) \approx R_{\text{zpo}}(\lambda) < 1 \tag{3}$$

for the reflection coefficients R_{phot} for photons and R_{zpo} for zero-point oscillations of wavelength λ. In the mid of the second page of his article [7], Casimir made a remark which is demonstrating that he shared assumption (3). We cite Casimir's remark on page 5 below equation (4). Assumption (3) is hardly tenable from the point of view of quantum-field-theory. The validity of the observed Casimir-force as a proof of the physical existence of zero-point energy has been discussed elsewhere in the framework of quantum-field-theory [9]. In the present article, assumption (3) is accepted as starting point of the evaluation, ignoring for the moment being concerns with regard to quantum-field-theory.

We retrace Casimir's computation in section 3, sticking to assumption (3), and consequently without the switch to infinite conductivity of the metals, which Casimir made at the end of the computation. The result of our computation is quite surprising: If (3) is true, then the experimental confirmation of (2) is proving — exactly opposite to Casimir's interpretation — that the electromagnetic field's zero-point energy does *not* exert forces onto metallic plates, i. e. that the observed forces by no means are related to the electromagnetic field's zero-point energy.

In section 4 we will raise objections against assumption (3), and point out inconsistencies, which undermine Casimir's arguments even more.

3. Computation with $0 < R_{\mathbf{zpo}}(\lambda) < 1$

Casimir considered a rectangular cavity-resonator of dimensions $X \times Y \times (Z + P)$, see figure 1 on the next page. There is a plate of thickness P inside the cavity, which is movable in Z-direction and aligned parallel to the cavity's XY-face. The distance between the plate and the side walls is D and $Z - D$, respectively.

We firstly consider the left cavity. At temperature T, it's walls and the plate are in thermodynamic equilibrium with the electromagnetic blackbody radiation within the cavity. The spectrum of

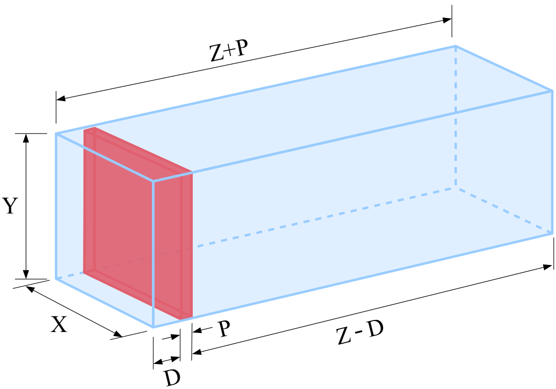

Fig. 1 : Cavity resonator with movable plate

the radiation's wave numbers is discrete:

$$k_{rst} = \sqrt{\left(\frac{r\pi}{X}\right)^2 + \left(\frac{s\pi}{Y}\right)^2 + \left(\frac{t\pi}{D}\right)^2}$$

with $r, s, t \in \mathbb{N}$ (4)

There are 2 modes each with $r, s, t = 1, 2, 3, \ldots$ and 1 mode each with one of the indices 0 and the both other indices $1, 2, 3, \ldots$ [10, chap. D.II.2.b.].

This equation does not hold for arbitrary wave numbers, because any metal becomes transparent for radiation of sufficiently high frequency. Casimir emphasized this fact in his publication [7]: "In order to obtain a finite result it is necessary to multiply the integrands by a function $f(k_{rst}/k_{\mathrm{M}})$ which is unity for $k_{rst} \ll k_{\mathrm{M}}$ but tends to zero sufficiently rapidly for $(k_{rst}/k_{\mathrm{M}}) \to \infty$, where k_{M} may be defined by $f(1) = 1/2$. The physical meaning is obvious: for very short waves (X-rays e. g.) our plate is hardly an obstacle at all and therefore the zero point energy of these waves will not be influenced by the position of this plate." Note that this remark of

Casimir is nothing but another wording for assumption (3), onto which our evaluation is built.

To design $f(k_{rst}/k_{\mathrm{M}})$ realistic, we assume the cavity and the plate to be made of copper. The reflectivity R as a function of wave number is quite complicated, see [11, fig. 1b]. For the purpose of our investigation, the rough approximation

$$R = \exp\{-\, k_{rst}/k_{\mathrm{M}}\} \quad \text{with } k_{\mathrm{M}} = 38 \cdot 10^6 \mathrm{m}^{-1} \tag{5}$$

for the reflectivity of copper is completely sufficient. (k_{M} is about $0.8\times$ the plasma-wavenumber of copper [11] according to the Drude-model.)

At temperature $T = 0$, only the zero-point oscillations of the electromagnetic field are excited. The energy per mode then is $\hbar c k_{rst}/2$, and the zero-point energy enclosed within the cavity is

$$U_0 = 2 \sum_{r,s,t=0}^{\infty}{}' \frac{\hbar c k_{rst}}{2} \exp\{-k_{rst}/k_{\mathrm{M}}\} \ . \tag{6}$$

The prime$'$ at the summation symbol is a reminder, that the multiplicity of polarizations, as indicated in (4), has to be considered. At most one of the numbers r, s, t of an oscillation mode can be zero, and terms with one zero index get a factor $1/2$.

Casimir considered the limit $X \to \infty$ and $Y \to \infty$. Thus he could replace the sums over the discrete indices r and s by integrals. Only the sum over t is still considered discrete. It has been elaborated elsewhere [12, sect. 4] in very detail, how (6) then can be transformed into

$$\begin{aligned} U_0 = \frac{\pi^2 \hbar c X Y}{2} \Bigg(& \frac{6 D k_{\mathrm{M}}^4}{\pi^4} - \frac{1}{360\, D^3} + \\ & + \sum_{j=6}^{\infty} \frac{B_j}{j!} \, \frac{(j^2 - 5j + 6)\, \pi^{j-4}}{k_{\mathrm{M}}^{j-4} D^{j-1}} \Bigg) \ . \end{aligned} \tag{7}$$

The coefficients B_j are the Bernoulli-numbers

$$B_0 = 1$$

$$B_j = -\sum_{n=0}^{j-1} \frac{j!}{n!(j-n+1)!} B_n \quad \text{for } j > 0 \; . \tag{8}$$

The expansion, which led to the series with the Bernoulli coefficients, does converge only for

$$D > \frac{1}{2k_\mathrm{M}} \overset{(5)}{=} 13.2\,\mathrm{nm} \; . \tag{9}$$

For smaller distance D between the movable plate and the cavity wall, (7) is not valid.

(7) is the zero-point energy enclosed within the left cavity of figure 1. The energy within the right cavity is found due to replacing D everywhere by $Z - D$. Then the force

$$F_\mathrm{ZPE} = -\frac{\mathrm{d}U_{0,\mathrm{total}}}{\mathrm{d}D} \tag{10}$$

acting onto the movable plate, which is caused by the different content of zero-point energy in the both cavities, can be computed. (The index $_\mathrm{ZPE}$ stands for zero-point energy.) With the approximation

$$\left(\frac{D}{Z-D}\right)^4 \ll 1 \; , \tag{11}$$

which is well justified for all experimental evaluations of the Casimir force, the result is

$$F_\mathrm{ZPE} = -\frac{\pi^2\hbar cXY}{2}\left(\frac{6(k^4_{\mathrm{M,left}} - k^4_{\mathrm{M,right}})}{\pi^4} + \frac{1}{120\,D^4} + \right.$$
$$\left. + \underbrace{\sum_{j=6}^{\infty} \frac{B_j}{j!} \frac{(-j^3 + 6j^2 - 11j + 6)}{k_\mathrm{M}^{j-4}\,\pi^{4-j}\,D^j}}_{A}\right) . \tag{12}$$

Here different values $k_{\mathrm{M,left}}$ and $k_{\mathrm{M,right}}$ have been inserted, considering the unavoidable differences in reflectivity due to surface contaminations, scratches, and the like. This will be discussed in the sequel. Casimir did *not* consider such possible differences. Instead at this point of the computation, he switched to the limit $k_{\mathrm{M,left}} = k_{\mathrm{M,right}} \to \infty$, which immediately led to his formula (2).

Note the change in Casimir's interpretation of his computation! Read again his remark, cited below equation (4) on page 5, on the penetration of short-wavelength zero-point oscillations through the metal plate. When he made that remark he clearly assumed, that firstly the plate was made of real metal (which is a reasonable assumption), and that secondly the dependence on wavelength is similar for the probability of the transmission of zero-point oscillations and for the transmission of photons (which is a quite strange assumption, which we will discuss in section 4). Now switching to idealized, perfectly reflecting material, Casimir considered the parameter k_{M} as a merely formal cut-off parameter, introduced only for the regularization of diverging integrals. From this point of view, the assumption $k_{\mathrm{M}} \to \infty$ is completely reasonable. But if the computation shall be of any relevance for the interpretation of experiments, which are not conducted with ideal materials of infinite conductivity, but with real metals, then the computation must be continued with the realistic parameter $k_{\mathrm{M}} = (5)$. We stick to this parameter in accord with Casimir's primarily assumption as cited below (4), and in accord with the assumption $0 < R_{\mathrm{phot}}(\lambda) \approx R_{\mathrm{zpo}}(\lambda) < 1$ made in (3). There we already announced, that we will challenge this assumption in section 4.

Skipping the last term, named A in (12), indeed is justified in most cases, because the Bernoulli-expansion is converging rapidly:

$$\frac{A}{1/(120\, D^4)} \le \begin{cases} 5\,\% & \text{for } D > 0.2\,\mu\mathrm{m} \\ 1\,\% & \text{for } D > 0.5\,\mu\mathrm{m} \end{cases} \tag{13}$$

But it was a misleading idealizing, when Casimir dropped the term $6(k^4_{\mathrm{M,left}} - k^4_{\mathrm{M,right}})/\pi^4$. This term is describing the fragile balance of two forces, which are tremendous in comparison to the Casimir-force $\sim 1/(120\,D^4)$. Using the realistic parameter $k_{\mathrm{M}} \overset{(5)}{=} 38 \cdot 10^6 \mathrm{m}^{-1}$, the ratio of these forces is

$$\frac{6k^4_{\mathrm{M}}/\pi^4}{1/(120\,D^4)} \approx \begin{cases} 2.5 \cdot 10^4 & \text{for } D = 0.2\,\mu\mathrm{m} \\ 1.5 \cdot 10^7 & \text{for } D = 1.0\,\mu\mathrm{m} \\ 9.6 \cdot 10^9 & \text{for } D = 5.0\,\mu\mathrm{m}\ . \end{cases} \tag{14}$$

Only with excellent, and actually not realistic match of reflectivities of the both sides of the plate, the Casimir force can become visible. For $D = 1\,\mu$m, the ratio of the forces is

$$\begin{array}{ccc} \dfrac{k_{\mathrm{M,left}} - k_{\mathrm{M,right}}}{k_{\mathrm{M,left}}} & & \dfrac{F_{\mathrm{Casimir}} = (2)}{F_{\mathrm{ZPE}} = (12)} \\ 10^{-4} & \Longrightarrow & 1.6 \cdot 10^{-4} \\ 10^{-5} & \Longrightarrow & 1.6 \cdot 10^{-3} \\ 10^{-6} & \Longrightarrow & 1.6 \cdot 10^{-2} \\ 10^{-7} & \Longrightarrow & 1.4 \cdot 10^{-1} \\ 10^{-8} & \Longrightarrow & 6.2 \cdot 10^{-1}\ . \end{array} \tag{15}$$

Actually, the experimentalists don't even try to achieve good matching surface conditions on the both sides of the movable plate. For example in the Purdue experiment [13, 14], the wall of the cavity is replaced by an Au-coated sphere, and the movable plate is replaced by a 3 μm thick silicon plate, which is Cu- or Au-coated on it's side facing the sphere, but un-coated bare Si on it's rear side. In a Yale experiment [15], the cavity wall again is replaced by an Au-coated sphere, while the movable plate is replaced by a SiN membrane, which is Au-coated on it's side facing the sphere, but un-coated bare SiN on it's rear side.

Still these experiments — like all other experiments, which as well don't pay attention to the surface conditions of their movable plate's rear sides — are seeing the Casimir-force (2), but not the force $F_{\text{ZPE}} =$ (12) caused by zero-point energy.

If the zero-point energy would exert forces onto metallic surfaces, then the term $k^4_{\text{M,left}} - k^4_{\text{M,right}}$ would dominate the observed forces in all of these experiments, making the tiny $\sim D^{-4}$ Casimir-force invisible. F_{ZPE} does not depend on D, and it can — depending on the relative reflectivities of the two sides of the movable plate — be attractive or repulsive. No force with that signature has ever been reported from any Casimir-force experiment.

Thus the experimental observations of the Casimir-force (2) do stringently prove, that the force $F_{\text{ZPE}} =$ (12) (caused by the electromagnetic field's zero-point oscillations) does not exist — if assumption (3) is correct. In the next section we will present reasons, why (3) probably is not correct.

4. $R_{\mathbf{zpo}}(\lambda)$ reconsidered

Equation (3) states an assumption on the reflection of photons and of zero-point-oscillations by metals. Photons can be reflected or absorbed or transmitted by metal plates. Does the same hold true for zero-point-oscillations? Can they as well be reflected, or absorbed, or transmitted by metals?

Absorption: It is obvious, that zero-point energy can not be absorbed by any real material. Otherwise that material, being surrounded by the electromagnetic field's ubiquitous zero-point oscillations, would absorb — even in case of an only tiny (but in any case finite) absorption coefficient — by and by short-wave-length zero-point radiation, convert it into high-energy phonons, which within soon would thermalize into many low-energy phonons. Eventually, the material would emit part of that energy again in form of low-energy photons. Thus matter would permanently convert high-energy zero-point oscillations into lower-energy phonons and photons, thereby heating up to infinity. As this is not observed in nature, one necessarily must assume that zero-point oscillations are never absorbed. Casimir commented nowhere in his article explicitly on the possible absorption of zero-point oscillations. At the very end of his computation, he switched to the assumption of perfectly reflecting metals, thus avoiding any discussion about absorption or transmission. But even before this final step, when in course of the computation he still assumed transmission of short-wavelength zero-point oscillations, he (correctly) never assumed that some zero-point oscillations might be absorbed.

Transmission: Casimir's clear statement with regard to the transmission of zero-point oscillations is cited below equation (4) on page 5. Thus Casimir assumed that zero-point oscillations behave similar to photons of same wavelength with regard to transmission through metals.

Reflection: As reflection, absorption, and transmission exhaust the possible outcomes of the interaction of photons and zero-point oscillations with metal plates, the probability for reflection is completely fixed by the probabilities for absorption and transmission. Casimir's assumption on transmission and the obvious assumption on absorption entail the assumption that (almost) no zero-point oscillations of sufficiently short wavelength are reflected, while most zero-point oscillations of long wavelength are reflected by metals. These considerations are in accord with the reflection coefficients (3), which again are based on the fact, that good fits of computations to the measured Casimir-force can be achieved under the assumption that the reflection spectra of metals are at least approximately the same with regard to reflection of photons and reflection of the (alleged) zero-point oscillations causing the Casimir-force [8].

One thus arrives at the following assumptions about the coefficients A of absorption, T of transmission, and R of reflection for phot = photons and for zpo = zero-point oscillations, respectively:

$$0 < A_{\text{phot}}(\lambda) < 1 \quad , \quad A_{\text{zpo}}(\lambda) = 0 \tag{16a}$$

$$0 < T_{\text{phot}}(\lambda) < 1 \quad , \quad T_{\text{zpo}}(\lambda) = A_{\text{phot}}(\lambda) + T_{\text{phot}}(\lambda) \tag{16b}$$

$$0 < R_{\text{phot}}(\lambda) \approx R_{\text{zpo}}(\lambda) < 1 \tag{16c}$$

This is a very strange, and in fact contradictory combination of assumptions, because — if considered on a microscopic scale — the processes of absorption and reflection of electromagnetic radiation are almost identical. If one assumes that the interaction between metals and photons and the interaction between metals and zero-point oscillations are quite similar with regard to reflection, then it is not reasonable to assume, that these interactions are fundamentally different with regard to absorption. Therefore it seems to be a far-fetched and not acceptable idea, that a real metal, which is able to reflect some zero-point electromagnetic radiation, should not be able to absorb that same zero-point radiation.

If one would nevertheless assume, that a real metal does reflect long-wavelength zero-point oscillations, and does transmit zero-point oscillations of sufficiently short wavelength, and still never absorbs zero-point oscillations of any wavelength, then of course our argumentation, as presented in the previous section, would need modification. Under this combination of assumptions, the reflectivity of the metal plate would be exactly identical for zero-point radiation (but not for photons!) impinging from either side, even in case of different surface platings. Then $k_{\text{M,left}} = k_{\text{M,right}}$ would hold for zero-point radiation (but not for photons!) in (12).

But (12) would still not reduce to Casimir's result (2), because the the gap-width D between the plates then would need a different computation: If a metal plate has a high-reflective coating on one side for radiation of some wavelength λ, but only low reflectivity on the other side, and if (16) should be correct, then most zero-point oscillations of wavelength λ would penetrate without attenuation from the low-reflective side through the metal, would be reflected at the coating of the other side, and would penetrate back again without attenuation through the metal. Thus the effective cavity length would be larger, if the low-reflective side of the plate is facing the opposite plate, and it would be smaller, if the high-reflective side of the plate is facing the opposite plate. As the thickness of the plate is of same order of magnitude, or in some experiments even much larger than the gap-width D between the plates, this effect should be measurable — in the unlikely case that it exists. Such experiments have not yet been published.

Much more plausible than the odd combination (16) of assumptions is the assumption, that metals either must be {able to reflect AND able to absorb} zero-point oscillations, EXOR must be {unable to reflect AND unable to absorb} zero-point oscillations (with AND and EXOR being logical operators, EXOR being the exclusive OR). In combination with assumption (16a), which can hardly be questioned, the conclusion becomes unavoidable that it is the second alternative, which is realized in nature. Note that the second alternative as well is confirmed by quantum-electrodynamics, according to which the electromagnetic field's zero-point oscillations do not interact at all with any real matter [9].

If consequently the zero-point oscillations are not reflected by metals, then of course all computations of the Casimir-effect with real metals become obsolete, because then the Casimir-effect is merely a nice theoretical construction which can work only with idealized plates, which are able to reflect zero-point oscillations, but not with any real metal. In the terminology of quantum field theory: The plates must be boundaries, but not tangible entities made from any real material. As experiments are conducted with real materials, Casimir's model consequently can not be of any relevance for the interpretation of these experiments.

This is no disaster, as a compelling alternative explanation for the attractive force between metal plates is available: The (retarded) van der Waals-interaction [16, 17]. Van der Waals-forces are caused by the interaction of polarizable matter due to the exchange of virtual photons, but not by some action of zero-point oscillations. Thus the Casimir-effect is no valid objection against the conjecture of Jordan and Pauli, "that in case of the electromagnetic field that zero-point radiation does not exist at all." [6]

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